A FOURIER APPROACH TO DIFFRACTION OF PULSED ULTRASONIC WAVES IN LOSSLESS MEDIA

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Abstract

Calculation of expected fields from pulsed sources is important in medical transducer design, source apodization studies, and physical acoustics. This paper presents a technique for calculating the diffraction field of a planar pulsed ultrasound wave with arbitrary spatial and temporal dependence based on Fourier domain techniques that are analogous to the techniques used in Fourier optics. The propagation is characterized by a total impulse response which is shown to be the Green's function. In the spatial frequency domain, the transform of the total impulse response is the propagation transfer function which behaves as a time-varying spatial filter of the form $J_0[\rho(c^2t^2-z^2)^{1/2}]H(ct-z)$. This spatial filter acts to decrease the relative content of the higher spatial frequencies as time progresses. The effect of the receiving aperture is easily incorporated in the Fourier domain as another spatial filter equal to the transform of the receiver's spatial sensitivity. Because of the use of the spatial Fourier transform the technique is amenable to computer implementation using FFT routines. Numerical examples are offered, including field calculations from circular and square piston sources, truncated Gaussian sources, and illustrations of the effect of receiving aperture size.

Introduction

While the propagation of monochromatic waves is well-solved by the application of the angular spectrum technique[1] or Fresnel integrals, the propagation of a pulse of ultrasound with arbitrary temporal and spatial shape is less well understood.

The approach followed here is based on the spatial impulse response method introduced by Stepanishen [2] and reviewed by Harris [3] where the field is expressed as a temporal convolution of the time excitation with the spatial impulse response of the propagation. It differs from Stepanishen's work in that linear systems theory is used to point out the importance of the point spread function (and its equivalence to the Green's function). Also, the expressions for the spatial impulse response functions are found in the spatial transform domain. In this domain, propagation of the wave is seen to be the application of a time-varying spatial filter to the spatial spectrum of the input wave.

Theory

The geometry is shown in Fig. 1. Given the z-directed velocity excitation over a rigidly baffled region of of arbitrary shape in the z=0 plane, we wish to find the acoustic velocity potential $\phi(x,y,z,t)$ at an arbitrary point in the positive-z half-space. We will assume that the z-velocity is given by

$$v_z(x,y,0,t) = T(t)s(x,y) \qquad (1)$$

In the impulse response technique, it has been shown [2,3] that the relation between the acoustic potential and the input z-velocity is

$$\phi(x,y,z,t) = T(t) \underset{t}{*} p(x,y,z,t) \qquad (2)$$

where the * symbol indicates convolution. We will call p(x,y,z,t) the **spatial impulse response**, defined as the velocity potential that will result when the source is excited by a z-velocity of the form $s(x,y)\delta(t)$ where $\delta(t)$ is the Dirac impulse function. Hence the problem of finding $\phi(x,y,z,t)$ is reduced to one of finding the spatial impulse response of the assumed spatial excitation.

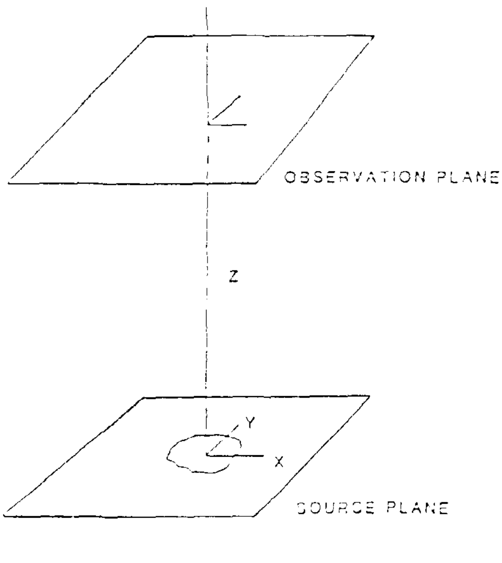

Fig. 1 Source and receiver geometry

To find the spatial impulse response we use linear systems theory [1, Chapt. 2]. The **total impulse response** $h(x,y,z,t)$ of a system is represented as in Fig. 2a. Here an impulsive input of the form $\delta(x,y)\delta(t)$ produces the total impulse response. If the system is linear and space-invariant (as is propagation in a linear homogeneous medium), then linear systems theory predicts that the response $p(x,y,t)$ to an arbitrary spatial excitation and an impulse temporal input of the form $s(x,y)\delta(t)$ is

$$p(x,y,z,t) = s(x,y) \underset{xy}{**} h(x,y,t) \qquad (3)$$

as shown in Fig. 2b. Hence to find the spatial impulse response, we need to find the total impulse response of the system $h(x,y,z,t)$.

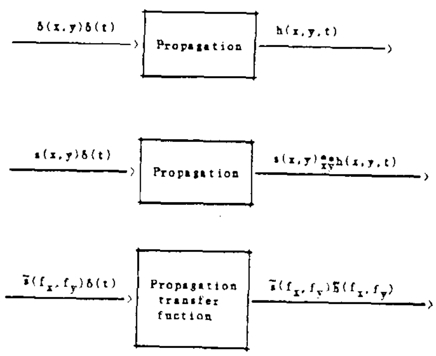

Fig. 2 a)Propagation impulse response
b) spatial impulse response
c) Propagation transfer function

The total impulse response of the system is the propagation field resulting from a source at the input plane of the form $\delta(x,y)\delta(t)$ that solves the wave equation and meets the boundary condition. This solution is just the Green's function satisfying the wave equation and boundary conditions. Hence, we find that if the Green's function is known, one knows the total impulse response.

The double spatial convolutions in Eq. 3 are difficult to implement on a computer. To convert the convolutions to multiplications, we enter the spatial frequency domain by taking the two-dimensional Fourier spatial transform of the system input and output. This is shown in Fig. 2c (where the tilde indicates the spatial Fourier transform of the function). The quantity $\tilde{h}(f_x,f_y,t)$ is the **propagation transfer function** and is the spatial transform of the total impulse response (or, equivalently, the spatial transform of the Green's function of the problem).

The Green's function for propagation in a lossless medium for the rigid baffle is known [4] to be (assuming only outward travelling waves),

$$g(x,y,z,t) = \frac{\delta(ct-R)}{2\pi R} \qquad (4)$$

where $R = (x^2+y^2+z^2)^{1/2}$. The spatial impulse response of this problem is

$$p(x,y,z,t) = 2s(x,y) \underset{xy}{**} \delta[t-(R/c)]/2\pi R \qquad (5)$$

We take the spatial transform of Eq. 4 to find the propagation transfer function $\tilde{h}_{p1}$ for lossless media as

$$\tilde{h}_{p1} = (1/\pi)J_0[\rho(c^2t^2-z^2)^{1/2}]H(ct-z) \qquad (6)$$

where $\rho=(f_x^2+f_y^2)^{1/2}$ and $H(\cdot)$ is the step function.

Since the results are going to be computer-implemented and normalized to maximum values, we will drop the multiplicative constants. From the preceeding discussion, we know that the spatial transform $\tilde{h}$ of the spatial impulse response is given by

$$\tilde{h} = \tilde{s}\tilde{h}_{p1} \qquad (7)$$

In this form we can identify the $\tilde{h}_{p1}$ term as a time-varying multiplicative spatial filter for the propagation in lossless media from a source in a rigid baffle. High spatial frequencies are relatively attenuated by this spatial filter. As time increases, the curve contracts causing a generally increased attenuation of the high spatial frequencies.

One finds the spatial impulse response for a given value of z in the following way. We calculate the spatial transform of the given $s(x,y)$ function, calculate the values of $\tilde{h}_{p1}$ at the same spatial frequencies for each value of time, and inverse spatial transform the product to produce the impulse response. Equation 2 is then used to find the desired acoustic velocity potential.

Numerical simulations

An additional advantage to the spatial spectrum approach discussed here beyond the physical interpretation of the propagation as a time-varying spatial filter is that the solutions are readily amenable to numerical solutions through the use of FFT routines and Fourier-Bessel algorithms. To illustrate numerical solutions, we consider some cases. The following simulations have been done using a 64x64 array of data. While the method gives a three-dimensional solution at any observation distance, one dimension is eliminated in the plots by representing the solution through a median of the source. The plots show the amplitude of the wave plotted against cross-direction and time. For plotting convenience, the following plots have been normalized to the maximum amplitude value. The width is normalized to the characteristic source size, D, (i.e., either the diameter or the width), and the time axis is normalized by the value of D/c. The origin of the time axis begins at z/c, the instant that the first part of the wave arrives at the observation plane. All plots are in an observation plane located 10 cm in front of the source plane.

1. Impulse excitation

Figure 3 shows the diffraction pattern from a circular transducer (the diameter is D=2.2 cm) excited by an impulse as observed on the axis, x=0. At t=z/c the potential is replica of the excitation. As time progresses, the potential is a combination of waves fron various points on the source. Late in time two distinct 'tails' are observed and were explained in terms of edge waves.

Figure 4 is a similar impulse excitation, but for a square transducer that is 2.2 cm on a side. The observation point is the same distance from the source.

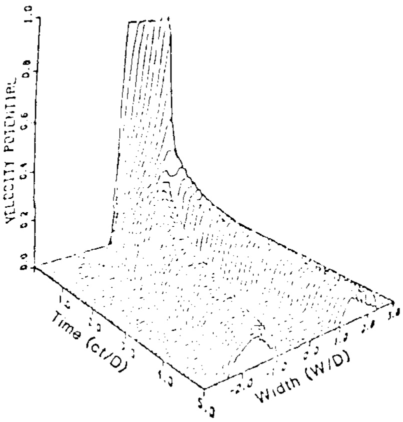

Fig. 3 Velocity potential for circular piston (D=2.2 cm), impulse excitation, z=10 cm

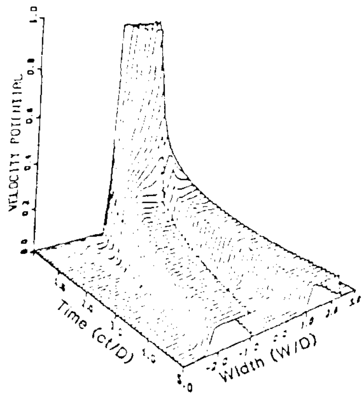

Fig. 4 Velocity potential for square transducer (D=2.2 cm), impulse excitation, z=10 cm

In Fig. 5, the pattern is shown for an axisymmetric gaussian shaped wave with an impulse time excitation. The 1/e point is $1.1/(5)^{1/2}$ cm from the center with the observation point kept the same as in the previous figures. The shape of the gaussian wave stays much the same because of the low spatial frequency content of this waveshape. Only large

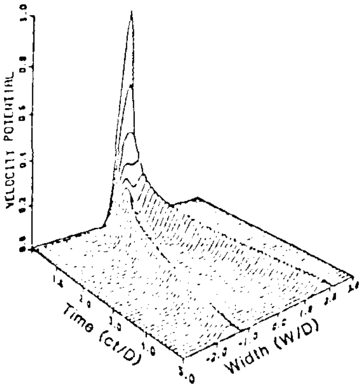

Fig. 5 Velocity potential from a truncated Gaussian wave (1/e point is 0.491 cm from center), impulse excitation, z=10 cm

values of time cause substantial spatial filtering for these low spatial frequencies.

2. Arbitrary time excitation

For a time excitation different than $\delta(t)$, the diffracted wave is a convolution as given by Eq. 2. Figure 6 is the circular transducer of Fig. 3 (the diameter is 2.2 cm) excited by a constant amplitude pulse of 10 microseconds duration for a lossless medium. The smoothing effect of the time-domain convolution is evident along the propagation direction.

3. Finite receiver effects

A receiver which is not a point receiver will perturb the observed field in the way that it averages the field. This averaging effect can be included in this method. The spatial frequency domain is well suited to include these effects

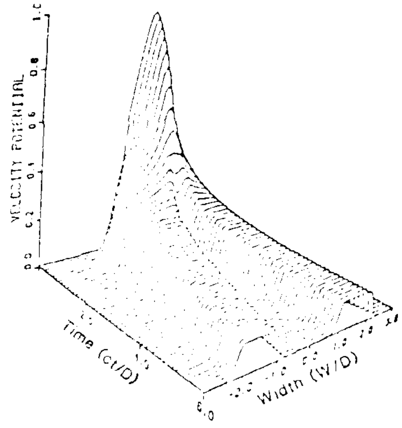

Fig. 6 Velocity potential of circular transducer for non-impulse excitation (Rectangular pulse excitation, T= 10^{-5} s.)

since the receiver contributes another low-pass spatial filter.

The averaged field can be written as the convolution of the field at the receiver and the receiver spatial sensitivity $A(x,y)$. Thus the receiving transducer is modelled in the spatial frequency domain as a multiplicative spatial filter given by $\tilde{A}(f_x,f_y)$. Figure 7 shows the detected field for a finite-sized circular receiver. The source is the circular transducer used in Fig. 3. The spatial convolution effect of the receiver is seen in the slight slope of the edge waves.

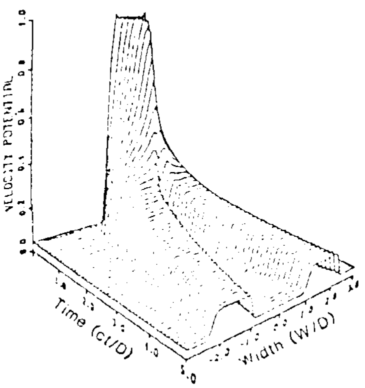

Fig. 7 Impulse response for finite size circular receiver(Receiver radius=1/2πcm). Circular source (D = 2.2 cm).

Summary

This paper presents a computationally efficient method of computing the transient acoustic waves in lossless and lossy media. The fields are expressed in terms of the spatial impulse response which is found by inverse transforming the product of the transform of the spatial excitation and the propagation transfer function. The propagation transfer function has been shown to be the transform of the Green's function which is the total impulse response of the propagation problem. No assumptions of the paraxial nature or restrictions on the propagation distance have been made. Additionally the solutions in the space domain use the computationally efficient Fourier transform. Once the spatial impulse response is known, Eq. 2 can be used to find the field for an arbitrary time excitation. The effects of a finite aperture receiver are also easily incorporated in the spatial frequency domain. Several numerical simulations have been given to illustrate the results of the technique.

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